PROPAGATION OF A JET OF VISCOUS LIQUID IN A MEDIUM CONTAINING A DENSITY JUMP

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The results of an experimental investigation into the laws governing the propagation of a jet of viscous liquid in a medium incorporating a density jump are studied for a Reynolds number range of $25 \leq R \leq 20 \cdot 10^3$. In addition to jets normal to the jump surface (vertical jets), horizontal jets travelling along the interface between the heavy and light liquids (jump surface) are examined. Photographs are presented, together with dynamic pressure measurements, illustrating properties of the jets studied which are unusual for a uniform medium: the extinction of turbulence, the existence of a limiting jet length, anisotropy of the jet, etc. An approximate explanation (within the framework of boundary-layer theory) is given for the effects in question.

1. In this investigation we studied a problem which has as yet received little attention: the propagation of laminar and turbulent jets in a medium containing a density jump. In the experiments the jump arose at the interface between two immiscible liquids arranged in a stable manner, that with the greater density (water) being at the bottom and that with the lower density (diesel oil, density 830 kg/m^3) at the top.

As a preliminary stage we used the same apparatus to study the propagation of liquid jets in a medium of another density (water jets in diesel fuel and conversely). Although the visual and photographic observation of the jets was greatly eased by choosing immiscible liquids, special experiments with miscible liquids (water and salt solutions of different densities) showed that the qualitative picture of the phenomenon remained the same. Of particular interest is the possibility of clearly visualizing the extinction of turbulence when the denser liquid (water) passes in jet form through the density jump. Despite the comparatively slight discontinuity in the densities, approximately equal to a ratio of 0.83, the photographs presented below indicate a substantial smoothing of the pulsations. There are also some obvious effects associated with the anisotropy of the jet structure, and so forth. A theoretical explanation for the observed phenomena and a quantitative estimation of the effects are given at the end of this article.

2. The experimental apparatus was an open rectangular trough with a length, width, and height equal to 470, 254, and 310 mm, respectively, made of Plexiglas. Water and diesel fuel were poured into the trough to form layers. At the bottom and in one of the sides of the trough were apertures containing tubes of diameter d = 2-4 mm for supplying liquid from the pressure tank. The position of the latter relative to the tube could be varied so as to obtain any specified conditions of outflow of the jet. The Reynolds number determined from the outflow parameters was varied over the range $25 \leq R \leq 20 \cdot 10^3$. This enabled experiments to be carried out with laminar, transient, and developed turbulent flow in the jet at the outlet from the nozzle.

In our experiments we measured the dynamic pressure (along the axis of the jet and over the cross sections) and also took ordinary and motion pictures of the jet. Dye was introduced into the liquid in order to make the flow readily visible. The dye was introduced into the supply tube under steady-state conditions of flow. This enabled us to determine the velocity distribution in the jet by measuring the velocity of the

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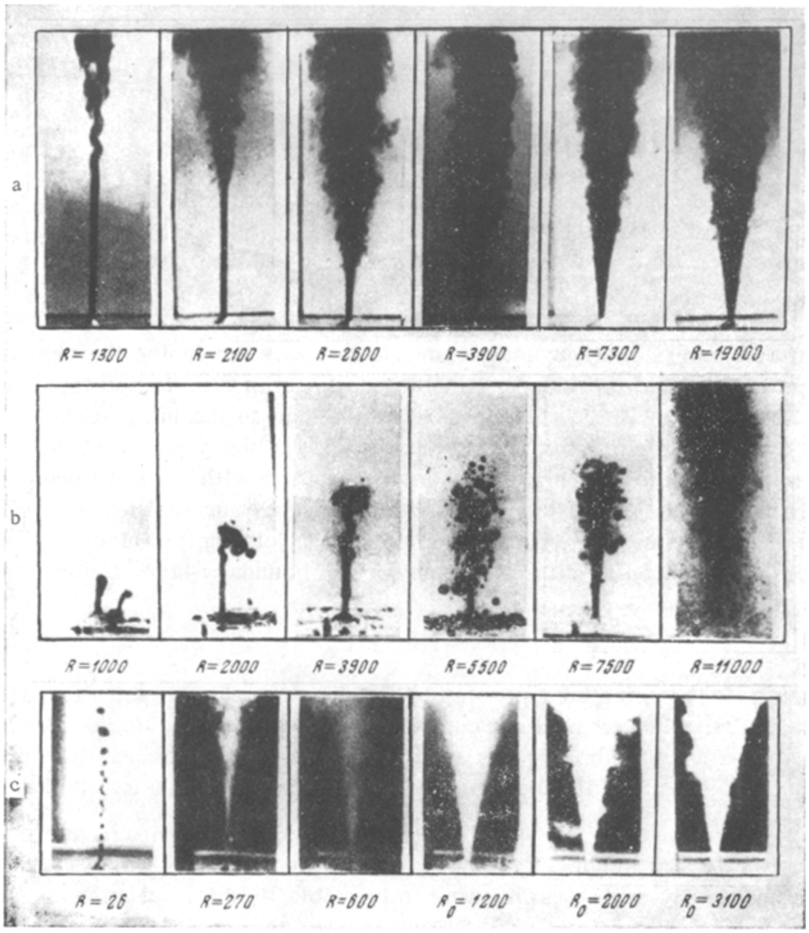

Fig. 1

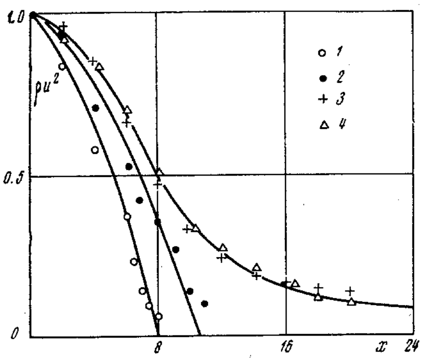

Fig. 2

dye front or leading edge (although the time of observation was restricted to a few seconds). We studied vertical jets, traveling in a direction perpendicular to the interface between the light and heavy liquids, and also horizontal jets generated in the neighborhood of the density jump. When studying the horizontal jets, we placed a mirror in the trough so as to record the horizontal and vertical projections of the jet at the same time. The ordinary and motion pictures were taken with Zorkii-M and SKS-1 M cameras, respectively. The dynamic pressure was determined with a Pitot tube 0.8 mm in diameter.

3. Figure 1 shows some photographs of liquid jets of various densities propagating in a homogeneous medium (*a* – water in water; b – water in oil; c – oil in water). Under the photographs are the Reynolds numbers R. The photographs show that for comparatively small values of the Reynolds numbers ($R < 3000$) three characteristic regions may be distinguished in the field of flow, corresponding to laminar (cylindrical part of the jet), transient (zone of sharp increase in perturbations), and developed turbulent flow [1]. As the Reynolds number R increases, the length of the cylindrical section shortens considerably and turbulization of the flow starts in the immediate neighborhood of the nozzle. When a jet of light liquid flows out into a denser one, turbulization of the flow takes place for considerably lower values of R. Hence, the developed turbulent mode sets in (for the same values of R) at a much shorter distance from the mouth of the nozzle, than when a jet of liquid with the same density as the surrounding medium is flowing. A qualitatively different picture is observed when a jet of heavy liquid propagates into a lighter one. In this case, for $R < 2500$ the forces of surface tension are sharply revealed – the photographs clearly show the formation of individual drops of heavy liquid. For large values

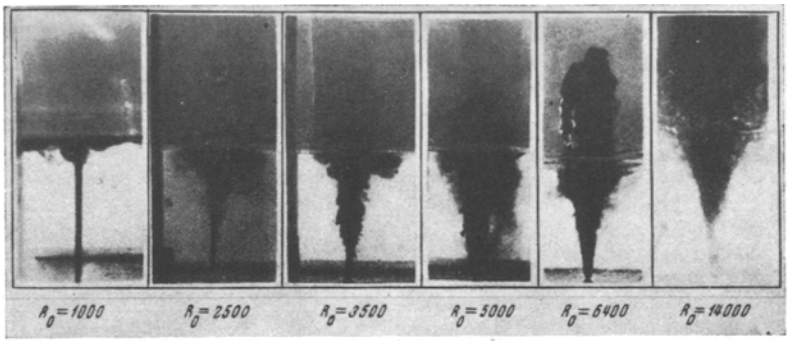

Fig. 3

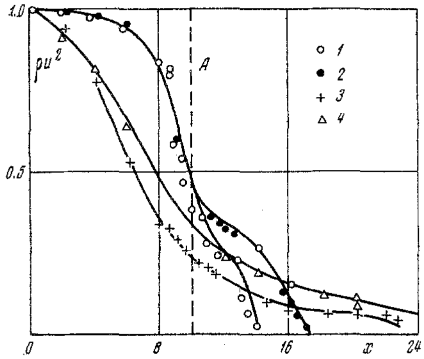

Fig. 4

of R, there is an intensive break-up of the drops, accompanied by the formation of a turbulent layer. As already noted, the qualitative picture remained intact when working with miscible liquids.

In cases of water flowing into water or oil into water, particularly in the region of the transition from laminar to turbulent flow (except for R = 25 in series b), the external form of the jet and the laws governing its development differed little from the propagation of gas jets. In contrast to this, when a water jet passes into oil at up to $R \le 10^4$ (and even higher when there is an unlimited depth of liquid in the trough), we observe the picture typical of the propagation of a jet in a direction opposite to the action of the mass force.

A characteristic feature of this type of flow is the limitation of the length of the jet to a certain ultimate value (which increases with increasing R). This effect – the existence of a limited length of development – is associated with the appearance of an Archimedes force directed against the motion induced by the jet. From the qualitative point of view the phenomenon is analogous to the so-called "annilation" of a jet of conducting liquid in a transverse magnetic field [2]. In both cases, the jet is retarded by a volume force (Archimedes or Lorentz) directed against the main flow. The results of our measurements of the dynamic pressure in jets of heavy liquid passing into a lighter liquid are shown in Fig. 2. The points 1, 2, 3, 4 represent values of R = 2880, 4000, 8000, and 18,000. These results show that for relatively low rates of outflow there is a sharp retardation of the jet. With increasing R, and hence, falling ratio of the Archimedes (negative) force to the force of inertia, there is an increase in the limiting length of development of the jet.

4. Figure 3 shows some photographs of vertical water jets traveling in a medium with a density jump (the interface between the liquids of different density, oil and water, lies at a distance of 10 diameters from the cutoff point of the nozzle). The experiments with vertical jets show that close to the interface, there is a considerable redisposition of the flow. For low rates of outflow, we find a reflection of the jet from the surface of separation; the jet spreads along the jump in the lower layer (outwardly reminiscent of the flow of a jet on a plane wall normal to the flow). For high rates of outflow, there is also a considerable slowing of the jet close to the interface, accompanied by a considerable expansion of the free boundary layer. The results of our measurements of the dynamic pressure along the axis of a jet traveling normal to the density jump are shown in Fig. 4. The points 1, 2, 3, 4 correspond to values of R = 2880, 4000, 8000, and 18,000. The vertical line A indicates the liquid interface. These results show that, in the neighborhood of the density jump, the profiles of ρu^2 are considerably deformed. At first (before the jump) there is a sharp fall in ρu^2 along the axis, and then, after passing through the interface, there is a considerable reduction in the intensity of mixing, accompanied by an extremely slight change in the dynamic pressure along the jet axis. This latter indicates laminarization of the flow due to the extinction of the turbulent pulsations in the zone of the jump. On increasing the rate of outflow, the characteristic inflection on the $\rho u^2 = f(x)$ curve degenerates; the distribution of the dynamic pressure along the axis becomes similar to the ρu^2 distribution in jets of constant density. (Actually, the inflection, i.e., the change in the rate of attenuation of the jet, only moves out to a great height.) Figure 5 illustrates some photographs of horizontal jets (view from the side and from below) propagating in a homogeneous medium (Fig. 5a), and in a medium with a density jump (Fig. 5b, c, d). We see from the photographs that when water flows into water (or oil into oil)

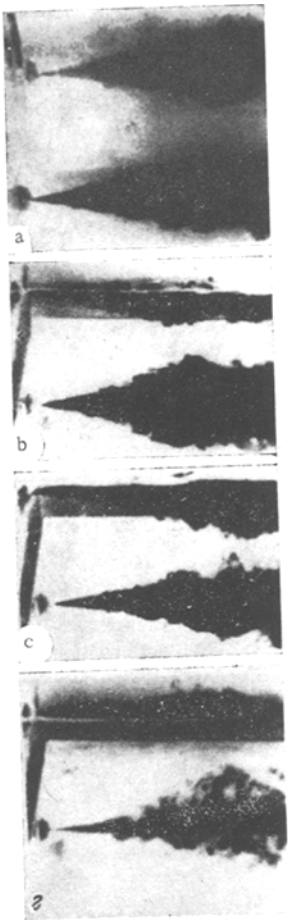

Fig. 5

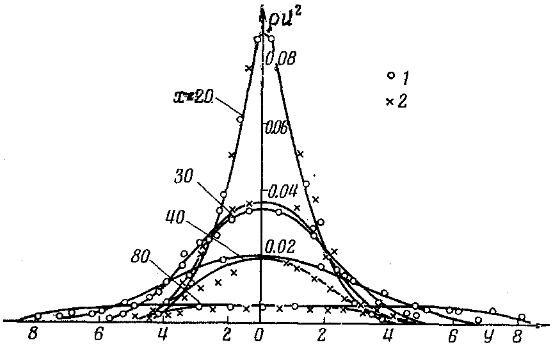

Fig. 6

the jet retains its ordinary axially-symmetrical form (the two projections, vertical and horizontal, almost coincide). When a water jet flows along the interface, there is a considerable deformation of the field of flow. When the axis of the jet lies in the plane of the interface, in particular, there is a considerable difference between the development of the jet in the horizontal and vertical directions – the width of the jet (parallel to the interface) greatly exceeds its thickness normal to the plane of the jump.

It is important to note the extremely complicated character of the changes taking place in the outer boundaries of the jet. In the vertical plane close to the mouth of the flow, there is at first a considerable broadening of the zone of mixing (some 50 diameters in length), the boundaries of the jet remaining almost rectilinear. After this there is a substantial curving of the boundaries of the jet, and also a certain contraction. Subsequently, at considerable distances from the cutoff of the nozzle, the thickness of the jet (along the vertical) remains almost constant. In contrast to this, the width of the jet (along the horizontal) increases monotonically, as in the case of flow into a homogeneous medium.

An analogous picture is observed in cases in which the axis of the nozzle lies slightly above or slightly below the interface (Fig. 5c, d). Here, also a difference arises in the broadening of the jet in the horizontal and vertical directions. In both cases, in the neighborhood of the interface, there is an extinction of the transverse vertical component of velocity, leading to a sharp disruption of the flow symmetry. This is confirmed by the results of our measurements of the total head in the jet cross sections (Fig. 6). Figure 6 shows the change in profiles ρu^2 in the horizontal (points 1) and vertical (points 2) projections of the jet propagating along the density interface. For this type of motion, there is a characteristic anisotropy of the flow, due to the extinction of the turbulent pulsations in the vertical direction. We note that the experimentally observed laws of development of the jet in the region of stable stratification of the medium are exactly the same for a number of different types of jet flow. In particular, they are characteristic for the flow in a medium behind a solid body [3-5].

5. Let us make a qualitative estimation of the effect due to the action of a volume force oriented against the flow. Let us write down the equations of motion and continuity for an axially symmetrical laminar jet of a heavy liquid traveling vertically upward:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y}\left(y\mu \frac{\partial u}{\partial y}\right) - g\Delta\rho \qquad (5.1)$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

Here, the last term in the first equation reflects the action of the Archimedes force due to the difference in the density of the liquid in the jet and in the surrounding medium ρ_0 ($\Delta\rho = \rho - \rho_0$). For simplicity, let us assume that the density of the outflowing liquid substantially exceeds the density of the medium, while the density distribution in the jet cross section is similar to the velocity profile ($\Delta\rho \sim \rho$, $\rho \sim u$). Under these conditions, we obtain the following integral relationship from (5.1)

$$\frac{dI}{dx} = -g \int_0^\infty \rho y\, dy, \quad I = \int_0^\infty \rho u^2 y\, dy \tag{5.2}$$

To this, we add the equation of conservation of the mass flow of the outflowing liquid, which corresponds to the usual assumption $\Delta\rho \sim \rho$

$$G = \int_0^\infty \rho u y\, dy \approx \text{const} \tag{5.3}$$

In the main part of the jet the solution of the original equations may be expressed in automodel form

$$u = u_m\, F(\varphi), \qquad y = \delta\varphi$$

We write Eqs. (5.2) and (5.3) in the following form:

$$\frac{dI}{dx} = -g u_m \delta^2 K_1, \quad I = u_m^3 \delta^2 K_3, \quad G = u_m^2 \delta^2 K_2 \tag{5.4}$$

where

$$K_n = \int_0^\infty F^n \varphi\, d\varphi \qquad (n = 1, 2, 3)$$

The last two equations enable us to express u_m and δ in terms of the momentum of the jet. Substituting these equations into the first of Eq. (5.4) and integrating this for an initial condition of $I = I_0$ at $x = 0$, we obtain the following for the jet momentum:

$$I / I_0 = \sqrt{1 - \chi} \tag{5.5}$$

where

$$\chi = 2g \left(\frac{G}{I_0}\right)^2 \frac{K_1 K_3}{K_2^2}$$

We see from this that for $\chi = 1$ the momentum of the jet vanishes. A qualitatively analogous result may be obtained for a turbulent (automodel) jet. In both cases a jet of denser liquid directed against the force of gravity is characterized by a finite limiting jet length, as experiment confirms. As already mentioned, this phenomenon is analogous to the retardation of a conducting liquid in a transverse magnetic field. From the physical point of view, it reduces to the exhaustion of the initial flow of momentum under the influence of the contrary mass force.

6. In order to explain the experimentally observed anisotropy, let us consider a turbulent jet traveling horizontally into a medium stably stratified in the vertical direction (along the z axis). In general, such a flow will be described by the equations of a three-dimensional turbulent boundary layer

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{\partial}{\partial y}\left[\nu_T \frac{\partial u}{\partial y}\right] + \frac{\partial}{\partial z}\left[\nu_T \vartheta(\mathrm{R}_i) \frac{\partial u}{\partial z}\right]$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{6.1}$$

where ν_T is the turbulent viscosity in a homogeneous medium (determined, for example, from the second Prandtl scheme; $\nu_T = u_m L$, u_m, L are the scale of the velocity and the effective width of the jet). The factor $\vartheta(\mathrm{Ri})$ allows for the influence of stratification on the vertical transfer of momentum, and is a function of the local Richardson number

$$\mathrm{Ri} = g \frac{\partial \rho}{\partial z} \Big/ \left(\frac{\partial u}{\partial z}\right)^2$$

The system of equations (6.1) is unclosed. We therefore, confine ourselves to a qualitative estimation of the laws governing the development of the jet under consideration along the vertical and horizontal. To this end, let us assume that the motion may to a first approximation be described by two independent equations of plane flow, separately described for the xy and xz planes

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left[\nu_T\frac{\partial u}{\partial y}\right],\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=\frac{\partial}{\partial z}\left[\nu_T\vartheta(\mathrm{Ri})\frac{\partial u}{\partial z}\right],\quad \frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{6.2}$$

Analysis of these equations carried out in the ordinary way [2, 6, 7] leads in the automodel region to the following estimate of the width of the jet in the horizontal (L_y) and vertical (L_z) directions

$$L_y\sim x,\qquad L_z\sim\int_0^x\vartheta(\mathrm{Ri})dx \tag{6.3}$$

It is furthermore assumed that the extinction coefficient of the pulsations $\vartheta(\mathrm{Ri}) = \vartheta(x)$ is a function of simply the longitudinal coordinate. It follows that the width of the jet increases without any limit in the horizontal direction (as in a homogeneous medium) on moving away from the mouth of the jet. In contrast to this the dimensions of the jet in the vertical direction assume a finite value at a certain distance from the mouth, and then remain constant. This is a consequence of the fact that, in the stably stratified medium, for a certain critical value of the local Richardson's number Ri, the coefficient ν_T vanishes (see [7], Part 1, Chapter 4), i.e., the turbulent momentum transfer in the vertical direction is completely suppressed. Thus, in a stably inhomogeneous medium the cross section of the initially circular turbulent jet acquires the shape of an ellipse with its minor semiaxis directed parallel to the direction of stratification. This result qualitatively agrees with the experimental data presented above. We note that the establishment of a finite jet size in the direction of the z axis is analogous to the degeneration of the jet flow of a conducting liquid in a transverse magnetic field. It is an important fact that the existence of a limiting size of the jet is only characteristic of turbulent flows, being due to the action of Archimedes forces on the pulsating motion. One result of this is the suppression of turbulent friction in one of the directions of expansions of the jet. Ultimately not only the tangential stress τ_{xz}, but also all terms in the turbulent stress tensor are suppressed. This should be observed particularly clearly in the limiting case of a density jump for which the local Richardson's number may exceed the critical value. It is this which explains the experimentally observed vanishing of the vortex structure in vertical jets traversing the interface between liquids of different densities (Fig. 3). Thus, the effects actually observed (the limiting jet length, the anisotropy of jet development, and the extinction of the turbulent pulsations of velocity) find a qualitative explanation within the framework of boundary-layer theory and the semiempirical theory of turbulence.

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